

## Introduction

- ▶ Visual filters can be modelled by derivatives  $G_k$  of the Gaussian function.
- ▶ The combined responses characterize the local structure of the image.
- ▶ This *Gaussian jet* representation is convenient because it is:
  - ▷ Steerable: Get any  $G_k(x, \sigma, \theta)$  from  $G_k(x, \sigma, \theta_1) \cdots G_k(x, \sigma, \theta_{k+1})$ .
  - ▷ Dimensionally separable, hence easily defined in 2D and 3D.
  - ▷ The natural code for typical image features (ridges, blobs, etc).
- ▶ But what about *complex cells*, cf. the Gabor 'energy model'?
- ▶ **Can the jet be made insensitive to small shifts of the image?**

## Replica Filters

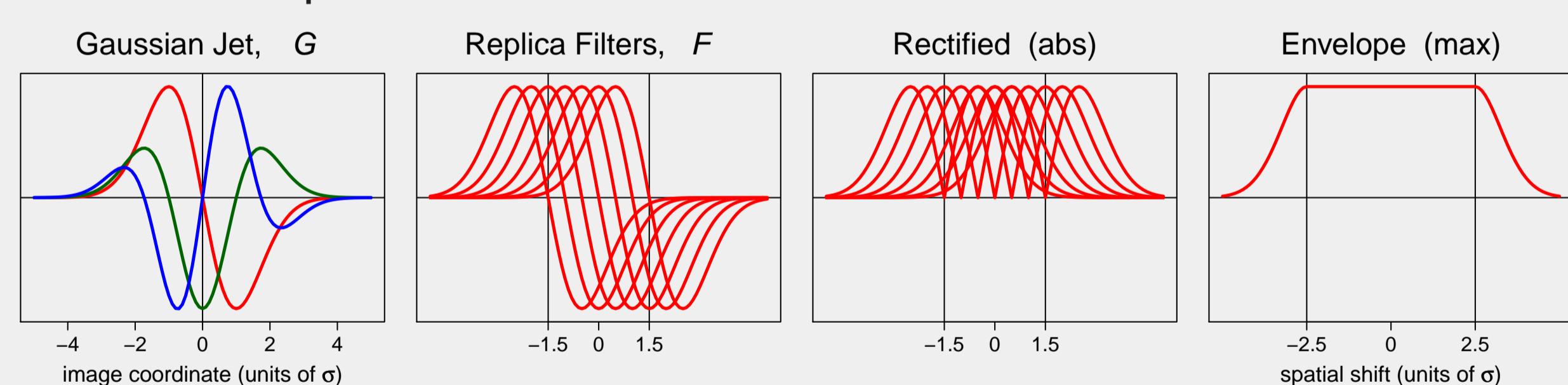
- ▶ Let  $F_\star(x, u)$  be a family of ideal filters, shifted by  $u$ .
- ▶ These can be Taylor-approximated from  $F_\star(x, 0)$  and its derivatives.
- ▶ In particular, choose the edge-templates  $F_\star(x, u) = G_1(x - u, \sigma)$ .
- ▶ Now define the **replica filters**,  $F \approx F_\star$ , from the  $D$ th-order jet:

$$F(x, u) = \sum_{k=0}^{D-1} \frac{-u^k}{k!} G_{k+1}(x, \sigma)$$

- ▶ Problems: Unstable, and nature of the approximation is unclear.
- ▶ Solution: Allow **polynomial** weights  $P_k(u)$ , and solve by least-squares.

## Impulse Response

- ▶ Schematic representation:



- ▶ More derivatives are needed in practice (see 'Filter Construction' box).
- ▶ The interval of approximation is  $\pm \rho \sigma$ , with  $\rho = 1.5$  here.
- ▶ Note that the family of replica filters is *continuous* (only 7 shown).

## Neural Implementation

- ▶ The linear-response vector is  $q_j = F_j \cdot s$ , one value for each shift.
- ▶ A neurally plausible 'soft-max' is used to compute the envelope:

$$q_\star = \sum_j w_j |q_j| \approx \max |q_j|$$

- ▶ The weights are defined by a **nonlinearity** and **normalization**:

$$w_j = \exp(\mu |q_j|) / \sum_j \exp(\mu |q_j|)$$

## Matrix Formulation

- ▶  $F$  : Replica filters (rows)       $P$  : Polynomials (columns)
- ▶  $G$  : Gaussian derivatives (rows)       $M$  : Monomials (columns)
- ▶  $s$  : Input signal (column)       $C$  : Estimated coefficients

- ▶ The defining equations are:

$$F = PG \quad \text{where} \quad P = MC$$

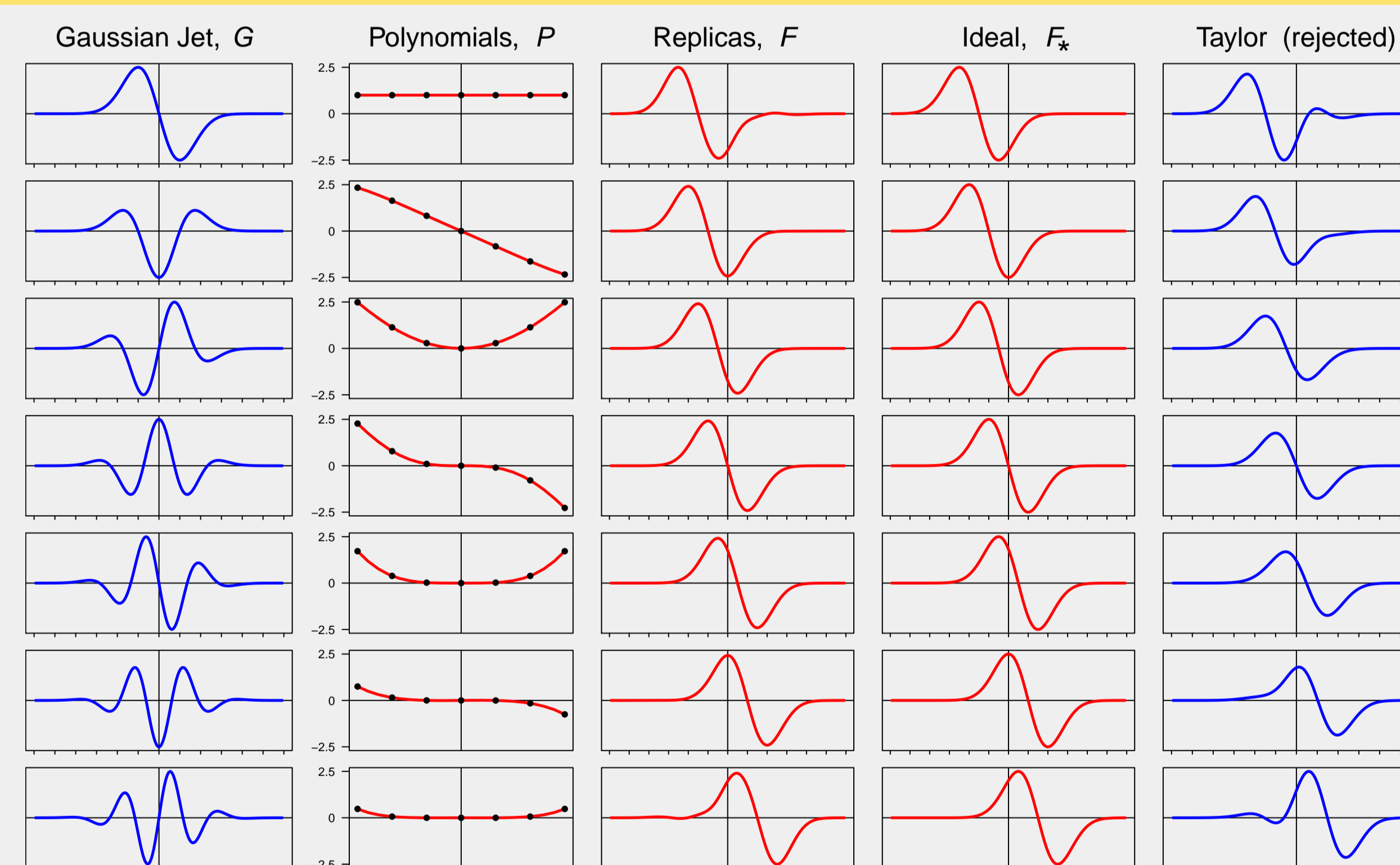
- ▶ The 'filter design' problem is to estimate  $C$ , given ideal filters  $F_\star$ .
- ▶ Least-squares solution is the pseudo-inverse of a Kronecker product:

$$\text{vec}(C) = (G^T \otimes M)^+ \text{vec}(F_\star)$$

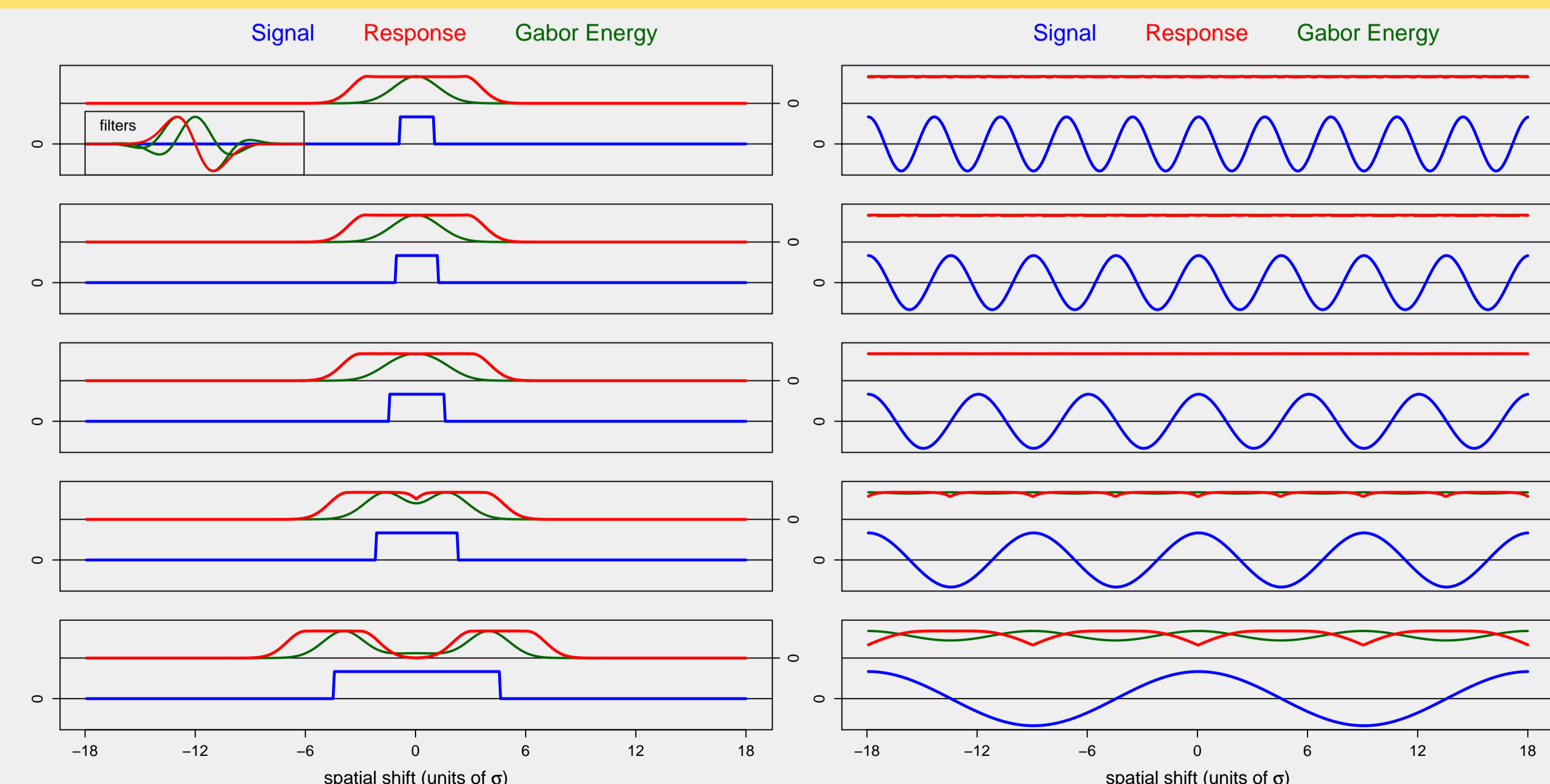
- ▶ The replica response  $q$  is a **linear transformation** of the jet response  $Gs$ :

$$q = Fs = P(Gs)$$

## Complete Example



## Bar and Grating Responses



## Two-Dimensional Filters

- ▶ The image gradient at 2D position  $x$ , and scale  $\sigma$ , is estimated as:

$$\nabla S(x) = [G_1(x, \sigma, 0) \cdot S, G_1(x, \sigma, \pi/2) \cdot S]$$

- ▶ The replica filters can find the local maximum of the gradient:

$$u_\star = \arg \max_{u \in \mathcal{F}} |\nabla S(x + u)| \quad \text{where} \quad \mathcal{F} = \{u : |u| < \rho \sigma\}$$

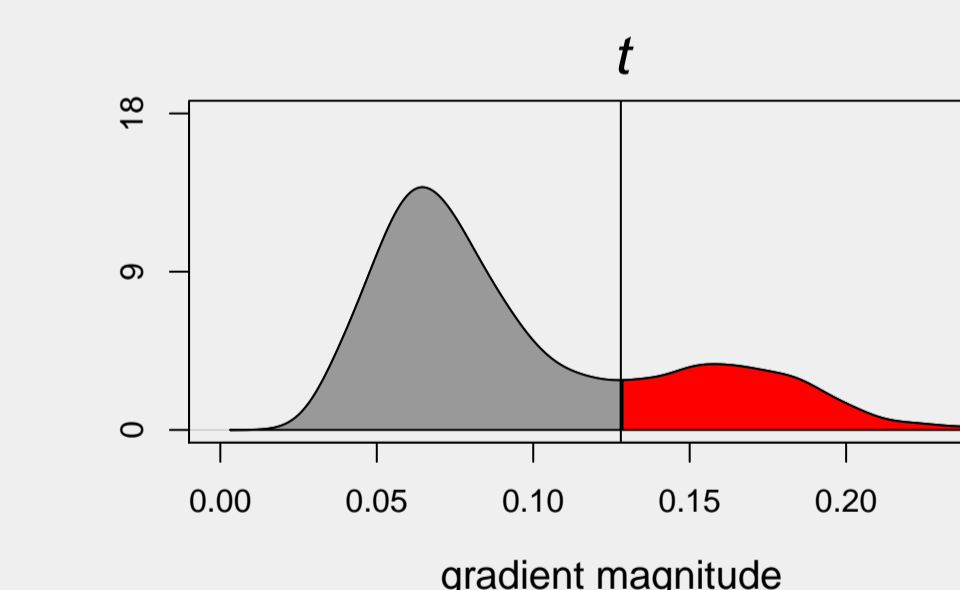
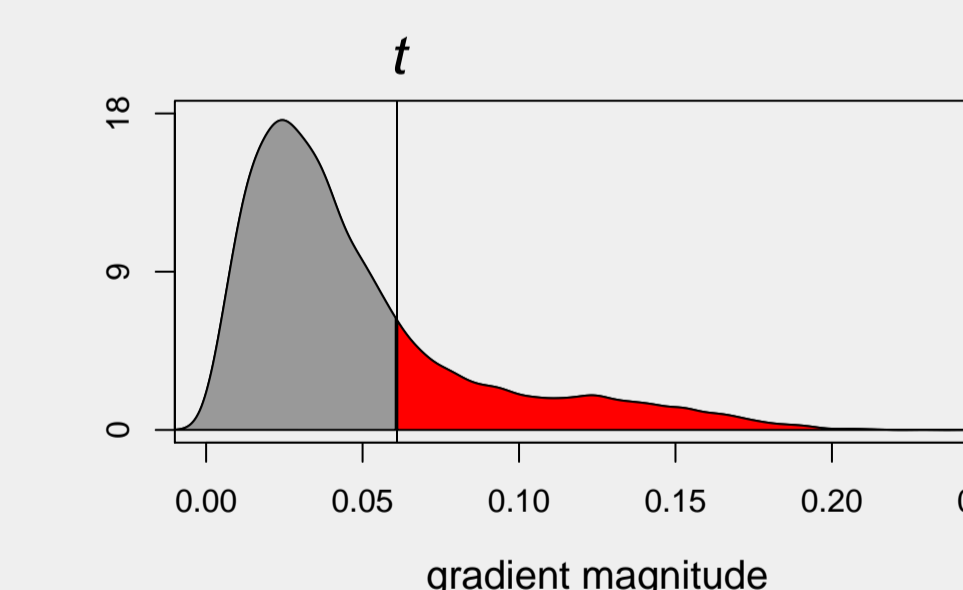
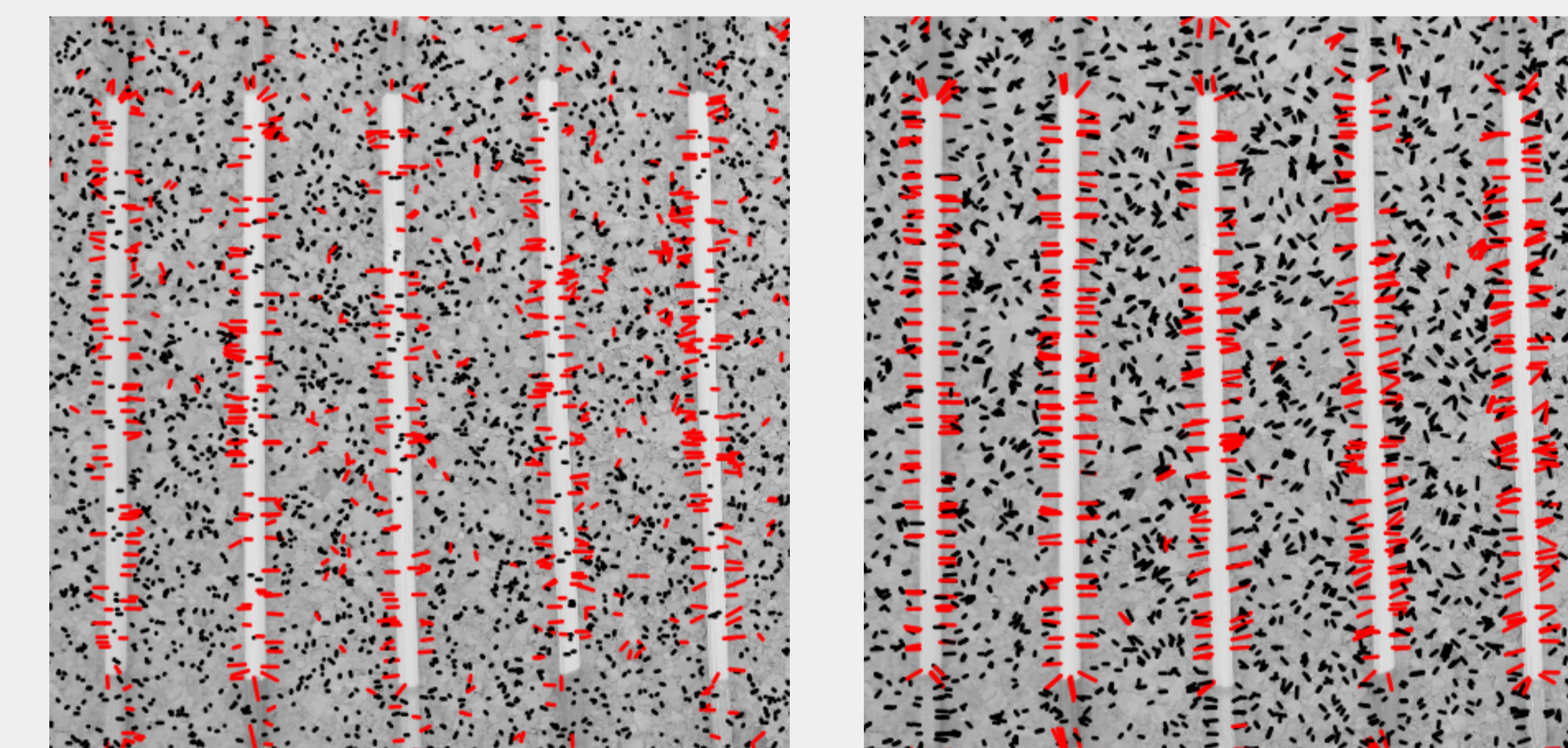
- ▶ The disk  $\mathcal{F}$  is the **receptive field** of the mechanism.

## Natural Image Experiments

- ▶ Make a coarse ( $\sim 1\%$  pixels) random sampling of the gradient.
- ▶ Compare  $\nabla S(x)$  to  $\nabla S(x + u_\star)$ , using  $\sigma = 3$  pixels and  $\rho = 1.5$ .
- ▶ Split each distribution at  $P(|\nabla S| < t) = 0.75$ ; plot strong vectors in red.

Raw Gradient  $\nabla S(x)$

Adjusted Gradient  $\nabla S(x + u_\star)$



- ▶ The undersampled structure is better represented by  $\nabla S(x + u_\star)$ .
- ▶ The true gradient distribution is bi-modal (boundaries plus texture).
- ▶ But the randomly-placed filters are unlikely to fall on the boundaries.
- ▶ If  $x$  is within  $\rho \sigma$  of an edge, then  $|\nabla S(x + u_\star)| \gg |\nabla S(x)|$ .

## Conclusions

- ▶ A shift-insensitive response can be obtained from the Gaussian jet.
- ▶ The signal structure can be represented **geometrically**.
- ▶ The new model is steerable, and works in any number of dimensions.
- ▶ High-order filters, as seen in neural data, are needed in the jet basis.