Cameras parameters initialization of 3D kinematic systems

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Abstract

3D information of the scene can be extracted from images acquired by cameras. Before the actual reconstruction camera calibration has to be done. Reconstruction accuracy is highly dictated by the calibration. Two typical demands, which are not easily simultaneously satisfied, are: calibration has to be done in fast and convenient manner and yet assure high degree of reconstruction accuracy. Computational part of calibration usually includes camera parameters initialization and refinement based on initial set of values. The goodness of initial set greatly affects refinement procedure in terms of convergence speed and ultimately reconstruction accuracy. This work proposes new calibration method for 3D kinematic systems. It shortens commonly used calibration procedure, gives better initial parameter values for refinement procedure which in turn is supposed to assure faster and safer convergence of iterative minimization algorithm. Additionally, it will be shown that even without parameter refinement proposed method gives more accurate 3D reconstruction output.

1. Introduction

The central goal of machine vision is to understand 3D world from 2D images [1]. As a rule, it requires model that describe how an image is formed by the camera, i.e. its lenses. Camera calibration in the context of three-dimensional machine vision is the process of determining the particular model parameters [2], [3]. Many types of modeling exists [4] and the most common is pinhole camera model, already used in photogrammetry for a long time [5], augmented with a means for compensating for lens distortions. In fact, one usually divides camera model parameters into internal ones and external ones. The first group is describing the internal camera geometric and optical characteristics. The second group models 3-D position and orientation of the camera coordinate system relative to a certain world coordinate system (extrinsic parameters).

Once the calibration of camera(s) is preformed 3D reconstruction can be done by simple triangulation principle [6]. 3D scene information can be used in variety of applications [7]: reverse engineering, robot navigation, extracting 3D structure, industrial part inspection, computer graphics animation, sport and medicine etc. The last one mentioned frequently assumes so-called biomechanical analysis of human movement [8], [9], [10], [11], [12], [13]. In this case basic information about subject position in space is usually upgraded with information about velocity and acceleration. Consequently, such 3D reconstruction system is called 3D kinematic systems. This paper considers in particular calibration of 3D kinematic systems and proposes a new method which is, to the author's knowledge, not yet implemented in popular today's 3D kinematic systems. To list few of them: [14], [15], [16], [17], [18], [19]. Proposed method shortens commonly used calibration procedure, gives better initial parameter values for refinement procedure which in turn assure faster and safer convergence of iterative minimization algorithm. Also, it will be shown that even without parameter refinement proposed method gives more accurate 3D reconstruction output.

The rest of the paper is organized as follows. First we briefly emphasize some practical aspects of calibration. Then in the section 3 we explain calibration procedure of the 3D kinematic system used in this work. Immediately after follows explanation of our approach. Section 5 demonstrates experimental results using common calibration method of today's many 3D kinematic system and our proposed one. Last section discuses acquired results and draws conclusions from it.

2. Problem statement

In many cases, the overall performance of the 3D kinematic system strongly depends on the accuracy of the camera calibration. Usually, the accuracy is inversely proportional to the complexity and effort user has to encounter during the calibration procedure. In terms of procedural complexity calibration methods can be put in three categories. Traditional approach takes advantage of some form of calibration object whose geometry in 3D space is very accurately known [20], [21], [22], [23].. In such cases calibration can be done very accurately however it requires (expensive) apparatus which may not be always easily manipulated. On the other hand one can reach out for the self(auto)calibration approach [24], [25], [26], [27]. Here, there is no calibration object involved whatsoever and one uses only the information contained within image taken by the camera(s). Namely, auto-calibration demands only sufficient number of point correspondences across the image set in order to obtain projective reconstruction. Then, based on certain constraints imposed on internal and/or external parameters we are able to calculate rectifying homography that we'll takes us from previously projective reconstruction to metric one (perhaps even Euclidean in case of known scale). Apart from obvious advantages the major disadvantages are cases where appropriate constraints on cameras cannot be imposed and/or certain camera configurations (movements) are degenerate and parameters cannot be recovered [28]. The third approach in terms of (dis)advantages lays somewhere in between the first two mentioned. It uses properties of the scene such as orthogonal or parallel lines [6], [29], [30]. It does not require accurate, but cumbersome 3D structure and does not suffer from most disadvantages typical to the self calibration methods. Thus, it seems appealing for employment in 3D kinematic system if we are able to provide such a suitable scene features with orthogonal/parallel lines.

3. Typical calibration procedure

The original 3D kinematic systems used traditional 3D object which very often needed to be moved around the calibration volume. Over the time more user friendly methods have been developed [31]. Nowadays the typical calibration procedure of many commercial 3D kinematic systems consists of two steps. First one includes positioning orthogonal triad of axes (three wands rigidly attached) on the ground, somewhere around the center of volume to be calibrated. Each axis of orthogonal triad has certain number of relatively easily detected markers on, which relative position is accurately known. Second step assumes so called wand dance. It requires from the user to walk around the volume and wave with the wand trying to cover as many different orientations and positions of the wand with respect to each camera of the system. Wand itself has minimum of two distinct markers on, which relative distance is accurately known as well. What *exactly* each step serves for in terms camera calibration is not thoroughly available in public, since majority of the systems have commercial value. Still from the amount of information available in public (particularly from the manufacturer which system has been used during this work – see below; and others claim similar) the following can be summarized. The first step, positioning of orthogonal triad, has purpose of establishing world coordinate system and, in terms of calibration, initializing the systems cameras parameters. The second step (wand dance) should, based on known wand length, refine previously calculated initial values on desired calibration volume, otherwise encompassed during wand dance.

Cameras parameter refinement generally involves iterative nonlinear minimization procedure, of a certain cost function, which in turn demands sufficiently good sets of initial values [32]. The closer the set of initial values to the true ones the better chances are for (faster) convergence to the global minimum. On the other hand poor initial set of solution may converge, if ever, to the point which largely deviates from true set of values and consequently 3D reconstruction with such camera parameters would be impaired.

As it will be shown in the next sections, the proposed calibration yields better initial sets of solution which after parameter refinement (left for future work) should give ultimately more accurate 3D reconstruction results. Besides, proposed methods converts two calibrations steps to only one.

4. Proposed method

In contrast to perhaps some other applications, conditions within 3D kinematic system operates can be relatively easily provided with scenes with orthogonal/parallel lines. Specifically, in this work orthogonal triad of axes was used during the wand dance instead of single wand. At the end of waving with the triad it was simply put on the ground where it remained just for few seconds, enough for the user to reach the PC station to quit image acquisition. In that case both sources information, typically obtained in two steps are provided: firstly, the world reference frame at desired location in space and secondly enough frames needed for further parameter refinement and/or, in our case, information to calculate initial cameras parameter values.

4.1. Extraction of initial camera parameters

Our approach of extracting initial value of cameras parameters starts with identification of image of the absolute conic (IAC). The complete description and properties of absolute conic, i.e. its image can be found elsewhere [33]. Here, will be reviewed only the basics.

The absolute conic is a conic on the plane at infinity consisting of points **X** such that

$$\begin{aligned} &\mathbf{X} = \begin{bmatrix} x & y & z & t \end{bmatrix} \quad t = 0 \\ &x^2 + y^2 + z^2 = 0 \end{aligned} \tag{1}$$

where points with t=0 are called points at infinity and its images are so called vanishing points **v**. Writing the first three components of point **X** separately as **d** the defining equation for the absolute conic within plane at infinity takes even simpler form:

$$\begin{aligned} &\mathbf{d} = \begin{bmatrix} x & y & z \end{bmatrix}^T \\ &\mathbf{d}^T \cdot \mathbf{d} = 0 \end{aligned} \tag{2}$$

Let us recall the decomposition of camera projective matrix **P**,

$$\mathbf{P} = \mathbf{K} \cdot \left[\mathbf{R} \,|\, -\mathbf{R} \cdot \mathbf{t} \right] \tag{3}$$

where $\mathbf{K}$ is matrix of cameras internal parameters, $\mathbf{R}$ and $\mathbf{t}$ represents cameras external parameters. The image point (i.e. vanishing point) corresponding to point at infinity as mapped by a camera with matrix $\mathbf{P}$ (2) (3) is then given by

$$\mathbf{v} = \mathbf{P} \cdot [\mathbf{d} \quad 0]^T = \mathbf{K} \cdot \mathbf{R} \cdot \mathbf{d} \qquad \textbf{(4)}$$

Solving (4) for $\mathbf{d}$ and combining with (2) gives:

$$\mathbf{v}^T \cdot (\mathbf{K} \cdot \mathbf{K}^T)^{-1} \cdot \mathbf{v} = \mathbf{v}^T \cdot \boldsymbol{\omega} \cdot \mathbf{v} \qquad \textbf{(5)}$$

Image point $\mathbf{v}$ is on the image of the absolute conic if and only if (5) is satisfied. Thus, the image of the absolute conic is a plane conic $\boldsymbol{\omega}$ represented by the matrix $(\mathbf{KK}^T)^{-1}$. Obviously cameras internal parameters are neatly embedded in IAC and once matrix $\boldsymbol{\omega}$ is found application of Cholesky decomposition on it would yield us matrix $\mathbf{K}$ itself.

It can be shown that angle α between two lines in 3D space can be found using the information about vanishing points $\mathbf{v_1}$ and $\mathbf{v_2}$ of those two lines and $\boldsymbol{\omega}$:

$$\cos\alpha = \frac{\mathbf{v}_1^T \cdot \boldsymbol{\omega} \cdot \mathbf{v}_2}{\sqrt{(\mathbf{v}_1^T \cdot \boldsymbol{\omega} \cdot \mathbf{v}_1) \cdot (\mathbf{v}_2^T \cdot \boldsymbol{\omega} \cdot \mathbf{v}_2)}} \qquad \textbf{(6)}$$

Conversely, if the angle between two lines is known we have a constraint on $\boldsymbol{\omega}$. Generally, above expression is quadratic for arbitrary angle between lines. However assuring that angles between lines is 90° will give us linear constraint.

$$\mathbf{v}_1^T \cdot \boldsymbol{\omega} \cdot \mathbf{v}_2 = 0 \qquad \textbf{(7)}$$

Matrix $\boldsymbol{\omega}$ is symmetric and homogenous. Without any further assumption about internal camera parameters (such as zero skew, known aspect ratio) we need minimum of five such orthogonal line pairs to find the exact solution for matrix $\boldsymbol{\omega}$. For more then five pairs a least square solution can be found. And in our case during the wand dance with orthogonal triad of axis that's exactly what is obtained. Vanishing points itself can be commonly found as intersection of parallel lines or, as it is in our case, from known ratios on single line (wand).

5. Experiments and results

3D kinematic system used during the course of this work, called Smart, is commercially available by the eMotion company [15]. The system version used (version 1.10, Build 2.39) consists of 9 cameras (50 Hz). It is so called optoelectronic system which actually reconstructs positions of passive retro reflective markers, attached to subject's point of interest. Markers are illuminated by stroboscopic IR sources of light attached to cameras itself and the cameras are additional equipped with IR filter. Smart is installed in Biomechanic laboratory of Peharec Polyclinic in Pula, Croatia [34]. The system is there used on daily basis for various motion analyses of healthy and injured subjects too. For more in-depth motion analysis the synchronized system add-ons are also two Kistler force platforms and 8 channels EMG device.

The first experiment consisted of typical system calibration as proposed by system manufacturer. Orthogonal triad of axes (each axis 60 cm long) was positioned on the floor. Each axis of triad defined one of the world coordinate axis and had certain number of retro reflective markers on. Vertical axis has 3 markers and two horizontal axes 4 and 2 each. Relative positions of markers are accurately known. Visual check was preformed that each cameras 'sees' all triad wands, i.e. markers on it and image acquisition was undertaken for a few seconds. Afterwards, as apart of second step orthogonal triad was removed and wand dance with a single wand was performed for another couple of minutes. The entire procedure was carried out by a trained polyclinic personal to ensure calibration results would not be perhaps impaired by the inexperience. Finally, Smart's software routines were started to compute cameras parameters based on acquired images from two steps of calibration.

Second experiment assumed our approach where wand dance started right away with the orthogonal triad of axis. It lasted roughly 60 seconds (the half of the time proposed by Smart for its wand dance with a single wand) and at the end the triad was simply put on the floor to set the origin of the coordinate system of the working volume. In both experiments volume size was approximately 3.2 m × 2.2 m × 2.0 m.

Among rather comprehensive analyzing software Smart has also capability to export/import various data into/from Matlab: 2D image data of markers centroids from acquired sequence, 3D reconstructed data of markers, camera projective matrices etc. Three things were exported for further analysis. 2D image data of markers on orthogonal triad put the floor which otherwise serves for camera parameter initialization in case of Smart calibration procedure. Then, camera projective matrices calculated by the Smart calibration procedure were exported and finally 2D image sequence of wand dance with orthogonal axes triad.

Table 1.Cameras internal parameters, initial values, Smart method

Focal length [pixels]		Skew factor	Principal point [pixels]	
X direction	Y direction			
751,9	387,0	12,7	400,0	175,3
755,6	381,0	18,3	330,3	233,5
676,6	343,3	7,4	272,3	200,5
704,1	367,4	6,4	320,7	171,3
765,2	399,0	5,0	335,9	170,2
746,8	386,4	19,0	376,9	215,3
691,0	354,6	9,2	285,3	164,5
672,7	343,3	14,4	297,2	142,7
672,8	348,2	10,3	292,1	156,2

Table 2. Cameras internal parameters, final values, Smart method

Focal length [pixels]		Skew factor	Principal point [pixels]	
X direction	Y direction			
727,7	375,9	0,0	349,4	153,7
723,8	374,6	0,0	304,4	145,4
723,9	375,2	0,0	290,3	138,4
724,3	374,7	0,0	325,4	140,0
724,2	375,1	0,0	347,7	137,0
724,8	375,0	0,0	349,9	143,4
719,2	371,8	0,0	328,9	134,7
730,1	377,2	0,0	350,7	133,4
715,9	370,5	0,0	345,0	138,6

Table 3. Cameras internal parameters, initial values, our approach

Focal length [pixels]		Skew factor	Principal point [pixels]	
X direction	Y direction			
724,8	376,2	0,0	376,9	128,0
749,7	390,0	0,7	321,9	136,0
743,4	390,7	-0,5	267,5	132,1
728,3	376,5	-0,3	346,9	133,5
715,6	373,8	-1,5	332,8	145,2
734,6	381,0	0,2	381,5	136,6
715,3	372,9	-2,3	274,4	126,3
704,4	370,2	0,9	308,0	122,2
723,3	373,4	0,5	341,9	136,2

Once exported into Matlab the following was calculated. Based on 2D image data of markers on orthogonal triad of axis and its known spatial relationship camera projective matrices were calculated which are supposed to be further processed by the Smart software to refine them after the wand dance. Next, from 2D wand dance image data of orthogonal triad vanishing points in each frame, for different cameras, were calculated from known length ratios of markers on. As explained before it enabled us to find IAC and consequently cameras initial set of internal parameters as proposed by our method.

Initial values of cameras internal parameters for Smart method, final values as provided by Smart and initial values from our approach are given for comparison in the Table 1, Table 2, Table 3 respectively. Each row represents values for one of the nine cameras.

Table 4. Mean length error between reconstructed and true wand length [mm]

Smart initial	Smart final	Propose d method
29,25	9,25	14,27
16,02	7,58	10,36
14,59	6,59	4,53
14,01	6,15	5,23
19,83	8,97	6,81
12,17	10,37	4,20
14,03	6,64	5,43
13,48	9,35	5,31
20,85	6,20	7,73
17,4	4,42	4,79
18,3	4,42	4,50
25,15	5,93	8,75
15,49	8,60	4,58
17,11	4,32	6,44
35,25	6,54	5,37
16,3	5,04	3,61
38,64	13,63	5,73
23,77	6,68	7,72
15,36	11,19	11,11
15,77	5,42	6,22
13,8	6,06	3,65
15,62	4,25	5,10
24,66	5,87	5,59
14,28	7,42	3,57
17,74	5,87	6,02
12,41	4,64	3,47
23,89	5,26	5,15
19	12,35	5,38
14,7	4,44	6,10
9,56	4,34	6,67
20,03	8,53	4,83
23,76	6,82	5,76
20,04	5,27	4,21
14,42	8,00	5,95
11,2	8,23	3,68
15,21	4,49	11,80

Let us consider each camera pair separately, 36 pairs out of 9 cameras. Once the internal parameters of cameras are known and given enough image correspondences fundamental matrix [35] can be calculated and therefore orientation of each camera with respect to each other can be extracted from [6]. Finally, the complete projective matrices for each camera pair can be constituted. Projective matrices originating from data in Table 1 and Table 3 were separately calculated in order to reconstruct the length of the wand during the wand dance. Furthermore, the same length was reconstructed using the (final) projective matrices provided by Smart and exported in Matlab. Mean error between reconstructed lengths of wand and true known value (45 cm) are given in columns of Table 4 for all 36 possible camera pairs. First, second and third column reflect data from Table 1, final Smart provided projective matrices and Table 3 respectively. All 3D reconstructions and length calculations are performed on distorted image coordinates and no distortion correction was undertaken

6. Discussion and concluding remarks

It is evident from comparison in Table 1, Table 2 and Table 3 that practically in all cases proposed method is right from the start with its initial values closer to the final ones obtained after the Smart refinement. The most likely reason for it perhaps lay in the fact that initial values in our approach (Table 3) are obtained after wand dance with orthogonal triad throughout the calibration volume, i.e. occupying almost all image area. On the other hand Smart sets orthogonal triad of axis on the floor and basically uses only one position. That single position occupies rather small amount of volume (image size) to calculate the initial values. Besides it is quite possible that in practice even that one position is (close to) degenerate with respect to certain camera. Degenerate in sense for example that markers on one triad wand are overlapping the other and software is unable to distinguish between different axes. In fact, when that happens user is notified to change the position of a triad on the floor. In our approach during the wand dense of triad it is quite likely also that in certain frames such situation occurs. However, since we are acquiring rather large number of frames, sweeping across the calibration volume, such frames are easily discarded and have no significance.

It has been already notified that closeness of initial solution to the final one highly determines speed of convergence and in large number of cases determines whether there will be any convergence or none at all. It is not rare in the practice that after Smart parameters refinement is done user is notified that calibration failed due to the fact that no convergent set of solution is obtained and calibration procedure has to be done again. Or giving the larger residuals of parameter refinement procedure user is indirectly suggested to redo calibration anyway. That's another issue that goes in favor of our approach which gives as initial estimates values closer to the final ones and starting with them is less likely that calibration procedure would have to repeated.

To test the quality of initial sets of parameters, perhaps the simplest, 3D reconstruction was undertaken: assuming linear camera model and no distortion correction. Furthermore, testing separate camera pairs gave insight of results consistency for various camera configuration set ups. As shown in Table 4 mean error of our approach is not only significantly better then when using Smart's initial, but it is very close to the case when final Smart's parameter sets are used. That strongly indicates, once again, that our initial set is very close to Smart's final one.

Let us underline that Smart's final sets of parameters are used also on distorted image data, using linear camera model. For completeness, we need to say that 3D reconstruction that Smart would normally output, after distortion correction on images, are in the order of millimeter, in case of camera pairs considered. In case where simultaneously would be used all available cameras for triangulation results are even better.

The exact procedure how Smart refines parameters, based on known wand length, is not known to the authors. Besides, an open question still remains what would be our wand length reconstruction results after parameter refinement. Answer to that question is left for future work. Still, closeness of our initial sets of solution to the Smarts final ones strongly suggests that should be at least about the same, if not perhaps better.

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