# **Binaural Hearing for Robots**

**Sound-Source Localization** 



# **Binaural Hearing for Robots**

- 1. Introduction to Robot Hearing
- 2. Methodological Foundations
- 3. Sound-Source Localization
- 4. Machine Learning and Binaural Hearing
- 5. Fusion of Audio and Vision

# 3. Sound-Source Localization

- 1. Time difference of arrival (TDOA)
- 2. Estimation of TDOA by cross-correlation
- 3. Estimation of TDOA in the spectral domain
- 4. The geometry of two microphones
- 5. Direction of arrival

Radu Horaud

- 6. Using more than two microphones
- 7. Embedding the microphones in a robot head
- 8. Learning a sound propagation model
- 9. Predicting direction of a sound with a robot head
- 10. Example of sound direction estimation

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# **The Basic Setup**



#### **Notations and Definitions**

We start by considering a simple situation: two microphones and one sound source:

- $M_1$  and  $M_2$ : 3D known positions of microphones  $m_1$  and  $m_2$ .
- S: 3D unknown position of the sound source
- *s*(*t*): **unknown** acoustic signal emitted by the source
- $x_1(t), x_2(t)$ : **observed** acoustic signals recorded with the two microphones

#### **Microphone Recordings**

$$x_{1}(t) = s(t - \tau_{1}) + n_{1}(t)$$
  

$$x_{2}(t) = s(t - \tau_{2}) + n_{2}(t)$$
  

$$\tau = \tau_{1} - \tau_{2}$$

- $\tau_1$  and  $\tau_2$ : times of arrival from the source to the microphone(s)
- $n_1(t)$  and  $n_2(t)$ : additive noise
- τ: time difference of arrival (TDOA)
   between the two microphones

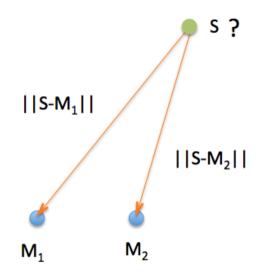
# **Direct Sound Propagation**

We assume a direct propagation path:

$$\tau_i = \frac{\|\boldsymbol{S} - \boldsymbol{M}_i\|}{\nu}$$

- $\|\cdot\|$  is the Euclidean norm
- $\nu = 344$  m/s is the speed of sound

#### Geometry



#### **Time Difference of Arrival**

• The signal reaches the two microphones with a delay:

$$egin{aligned} & au = au_1 - au_2 \ & = rac{\|oldsymbol{S} - oldsymbol{M}_1\|}{
u} - rac{\|oldsymbol{S} - oldsymbol{M}_2\|}{
u} \end{aligned}$$

- $\tau$  is the:
- TDOA (time difference of arrival) or
- ITD (interaural time difference).

# **Session Summary**

- Basic setup: two microphones and one source.
- Binaural recording with the direct propagation model.
- Geometric representation.
- Time difference of arrival (TDOA).

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#### **Estimation of TDOA**

The TDOA can be estimated via **cross-correlation** (CC) between the two microphone signals:

$$\hat{\tau} = \arg\max_{t'} \sum_{t=t_1}^{t_2} x_1(t) x_2(t-t')$$

 $\hat{\tau}$  is an estimation of the **true value**  $\tau$ .

#### **Robust Estimation of TDOA**

The normalized cross-correlation (NCC) offers robustness with respect to a gain difference between the two signals:

$$\hat{\tau} = \underset{t'}{\operatorname{argmax}} \frac{\sum_{t=t_1}^{t_2} x_1(t) x_2(t-t')}{(\sum_{t=t_1}^{t_2} x_1^2(t))^{1/2} (\sum_{t=t_1}^{t_2} x_2^2(t))^{1/2}}$$

The TDOA can also be estimated with the **generalized cross-correlation** (GCC) that offers more flexibility.

# **Estimation of TDOA Using GCC**

- The short-time Fourier transform (STFT) is applied to the two signals.
- The two signals may (or may not) be filtered.
- The cross power spectral density (cross-PSD) is estimated at each frequency.
- The TDOA corresponds to the phase offset that maximizes the cross-PSD over all the frequencies.

#### **Short-time Fourier Transform (STFT)**

• STFT of signal *x*(*t*) at **frame** *I* has a complex value for each **frequency** *f*:

 $X(1, l), \ldots, X(f, l), \ldots, X(F, l)$ 

- This is a complex-valued vector of dimension *F*.
- For each frequency f and frame I, X(f, I) is a complex number:

$$X(f, l) = \mathcal{X}(f, l)e^{\alpha} = \mathcal{X}(f, l)(\cos \alpha + j \sin \alpha)$$

- $\mathcal{X}(f, I)$  is the **magnitude** and
- $\alpha$  is the **phase**.
- $j^2 = -1$ .

#### **Microphone Signals in the Spectral Domain**

• STFT vector of the left-microphone signal  $x_1(t)$ :

 $X_1(f, I), f = 1 \dots F$ 

• STFT vector of the right-microphone signal  $x_2(t - t')$ :

$$X_2(f, I)e^{\alpha(t')}, f = 1 \dots F$$

- This results from the time-shifting theorem.
- *t*<sup>'</sup> corresponds to the unknown TDOA.

# **Cross Power Spectral Density**

$$\Phi_{x_1,x_2}(1) = \frac{1}{L} \sum_{l=1}^{L} X_1(1,l) X_2^*(1,l)$$
  

$$\vdots$$
  

$$\Phi_{x_1,x_2}(f) = \frac{1}{L} \sum_{l=1}^{L} X_1(f,l) X_2^*(f,l)$$
  

$$\vdots$$
  

$$\Phi_{x_1,x_2}(F) = \frac{1}{L} \sum_{l=1}^{L} X_1(F,l) X_2^*(f,l)$$

# **Session Summary**

A number of important concepts were addressed:

- Cross-correlation in the temporal domain
- The short-time Fourier transform of a microphone signal
- The TDOA in the spectral domain
- Cross-power spectral density of two signals

# 3. Sound-Source Localization

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#### **Cross-Correlation in the Frequency Domain**

A time delay corresponds to a phase shift in the spectral domain:

$$R_{x_1,x_2}(t') = \sum_{f=1}^{F} \Phi_{x_1,x_2}(f) e^{\alpha(t')}$$

**TDOA** estimation:

$$\hat{ au} = \operatorname*{argmax}_{t'} R_{x_1,x_2}(t')$$

#### **Prefiltering the Microphone Signals**

Convolution between signals and filters:

$$y_1(t) = h_1(t) \star x_1(t)$$
  
 $y_2(t) = h_2(t) \star x_2(t)$ 

In the spectral domain, for each frequency *f* and frame *l*:

 $Y_1(f, l) = H_1(f)X_1(f, l)$  $Y_2(f, l) = H_2(f)X_2(f, l)$ 

#### **Generalized Cross-Correlation**

$$\Phi_{y_1,y_2}(f) = \frac{1}{L} \sum_{l=1}^{L} H_1(f) H_2^*(f) \Phi_{x_1,x_2}(f)$$
$$R_{y_1,y_2}(t') = \sum_{f=1}^{F} \Phi_{y_1,y_2}(f,t') e^{\alpha(t')}$$

**TDOA** estimation:

$$\hat{ au} = \operatorname*{argmax}_{t'} R_{y_1,y_2}(t')$$

# **Filter Design**

 $\Psi(f) = H_1(f)H_2^*(f)$  is the frequency weighting. We have:

• The Smooth COherent Transform (SCOT):

$$\Psi(f) = (\Phi_{x_1, x_1} \Phi_{x_2, x_2})^{-1/2}$$

• The PHAse Transform (PHAT):

$$\Psi(f) = |\Phi_{x_1, x_1}|^{-1}$$

#### **TDOA Estimation with PHAT**

The PHAT cross-correlation:

$$R_{y_1,y_2}(t') = \sum_{f=1}^{F} \frac{\Phi_{x_1,x_2}(f)}{|\Phi_{x_1,x_1}(f)|} e^{\alpha(t')}$$

The TDOA estimation:

$$\hat{ au} = \operatorname*{argmax}_{t'} R_{ extsf{y}_1, extsf{y}_2}(t')$$

# **Session Summary**

- Cross-correlation in the frequency domain
- Signal filtering
- Generalized cross-correlation
- Choosing a filter
- TDOA estimation in the spectral domain

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#### Finding the Position of the Sound Source

The position of the sound source is given by solving the equation:

$$\hat{\tau} = \tau(\mathbf{S})$$

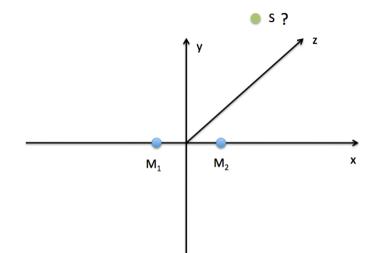
In more detail:

$$\hat{\tau} = rac{\|m{S} - m{M}_1\| - \|m{S} - m{M}_2\|}{
u}$$

Let  $M_1 = (-m, 0, 0)$  and  $M_2 = (m, 0, 0)$ .

S = (x, y, z) is the unknown source location.

# Finding the Position of the Sound Source



#### Finding the Position of the Sound Source

The equation develops as:

$$\nu\hat{\tau} = \left((x+m)^2 + y^2 + z^2\right)^{1/2} - \left((x-m)^2 + y^2 + z^2\right)^{1/2}$$

which be written as:

$$\frac{x^2}{\frac{\nu^2\hat{\tau}^2}{4}} - \frac{y^2}{\frac{4m^2 - \nu^2\hat{\tau}^2}{4}} - \frac{z^2}{\frac{4m^2 - \nu^2\hat{\tau}^2}{4}} = 1$$

#### **Two-Sheet Hyperboloid** $\mathcal{H}$

This equation has the following structure:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{b^2} = 1$$

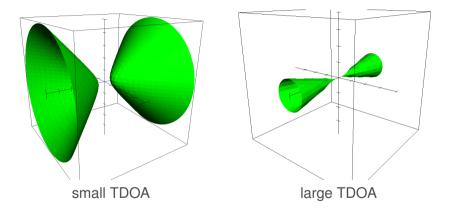
This is a *two-sheet hyperboloid*,  $\mathcal{H}$ , with foci in  $M_1$  and  $M_2$ . The sound source S = (x, y, z) should lie on  $\mathcal{H}$ .

#### Where is the Sound Source?

An estimated time difference of arrival  $\hat{\tau}$  defines a surface  $\mathcal{H}_{\hat{\tau}}$ . The surface exists only if:

$$0 < |\hat{\tau}| < \frac{\|M_1 - M_2\|}{
u}$$

# **Two-Sheet Hyperboloids**



# **Choosing a Sheet**

- The hyperboloid has to sheets (branches), one with x < 0 and one with x > 0.
- The source can be on either one of these two branches. From the above equations, we can write:

$$x = \frac{\nu \hat{\tau} \left( 2 \| \boldsymbol{S} - \boldsymbol{M}_1 \| - \nu \hat{\tau} \right)}{4m}$$

The sign of *x*, that depends on  $\hat{\tau}$ , selects one of the two sheets.

# **Session Summary**

- The TDOA is combined with the geometry of two microphone and an emitting source
- For any two given microphones, the TDOA has lower and upper bounds
- The source is "somewhere" onto a two-sheet hyberboloid

# 3. Sound-Source Localization

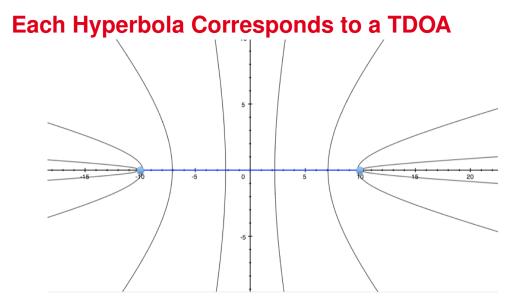
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#### **The Azimuthal Plane**

We consider an *azimuthal* plane, namely a plane that contains the two microphones, namely the plane y = 0:

$$\frac{x^2}{a^2} - \frac{z^2}{b^2} = 1$$

There is a hyperbola for each value of the TDOA



### **Direction of Arrival**

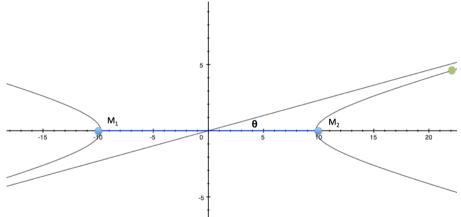
The slopes of the two asymptotes of the hyperbole are

$$an heta = \pm rac{b}{a} = \pm rac{\sqrt{4m^2 - (
u \hat{ au})^2}}{
u \hat{ au}}$$

The value of  $\hat{\tau}$  selects one of the branches (x > 0 or x < 0).

The sign of the slope selects front of back side.

### **Direction of Arrival**



# **Session Summary**

- The space is projected onto the azimuthal plane
- In this plane the source lies on a two-branch hyperbola
- The sound direction (azimuth) can be estimated from the asymptote
- One has to solve the left-right and front-back ambiguities.

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### **Increasing the Number of Microphones**

- The time difference of arrival (TDOA) must be robustly estimated
- The TDOA is estimated using GCC (generalized cross-correlation)
- The geometry determines the sound direction in the azimuthal plane
- What about increasing the number of microphones?
- Can we find the sound location in 3D?

### **Microphone Arrays**

 Consider *M* non coplanar microphones (*M* ≥ 4). There is one equation for each microphone:

$$x_i(t) = s(t - \tau_i) + n_i(t), \quad \forall i, 1 \le i \le M$$

• as well as a TDOA between any microphone pair:

$$\tau_{i,j} = \tau_i - \tau_j$$

### How Many Hyperboloids?

• For any three microphones *i*, *j*, *k*:

$$\tau_{i,j} = \tau_{i,k} + \tau_{k,j}$$

• Consequence: Only M - 1 pairs are needed among all the M(M - 1)/2 microphone pairs.

#### **M Microphones in General Position**

$$\nu \hat{\tau}_{1,2} = \| \boldsymbol{S} - \boldsymbol{M}_1 \| - \| \boldsymbol{S} - \boldsymbol{M}_2 \| \\
 \vdots \\
 \nu \hat{\tau}_{1,M} = \| \boldsymbol{S} - \boldsymbol{M}_1 \| - \| \boldsymbol{S} - \boldsymbol{M}_M \|$$

This system of equations has a unique solution **S** for a set of TDOAs  $\hat{\tau}_{1,2}, \ldots, \hat{\tau}_{1,M}$ 

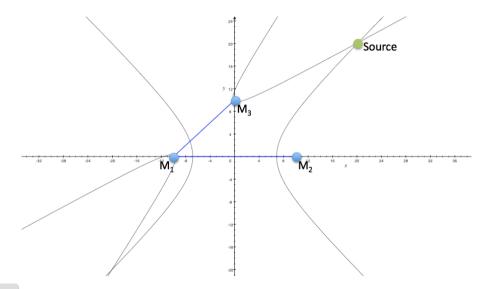
### Example: Three Planar Microphones

- $M_1 = (-m, 0, 0), M_2 = (m, 0, 0), M_3 = (0, m, 0)$
- We obtain two hyperboles:

$$\nu \hat{\tau}_{1,2} = \| \boldsymbol{S} - \boldsymbol{M}_1 \| - \| \boldsymbol{S} - \boldsymbol{M}_2 \|$$
$$\nu \hat{\tau}_{1,3} = \| \boldsymbol{S} - \boldsymbol{M}_1 \| - \| \boldsymbol{S} - \boldsymbol{M}_3 \|$$

• The time delays are estimated using GCC (generalized cross-correlation) applied to the microphone pairs (1,2) and (1,3).

# **Example With Three Microphones**



# **Session Summary**

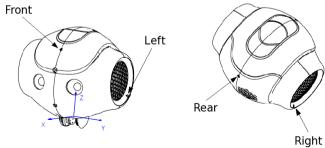
- A microphone array is composed of four non-coplanar or three planar microphones
- Each microphone and TDOA forms a two-branch hyperboloid
- The sound source is at the intersection of these hyperboloids
- There are two solutions: positive and negative

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### **Microphones Embedded in a Robot Head**





### Sound Localization with the NAO Robot

NAO has four non-coplanar microphones

The microphones signals are corrupted by various sources of noise:

- environmental noise
- reverberations
- robot noise (motors, fans, etc.)

A robust localization method is needed.

### **Far-field Assumption**

The sound source, *S* is parameterized by its direction with respect to the robot head:

- Euclidean coordinates: *x*, *y*, *z*.
- Spherical coordinates:  $r \sin \alpha \sin \beta$ ,  $r \sin \alpha \cos \beta$ ,  $r \cos \alpha$
- We make a **far field** assumption: *r* is large with respect to the head size:

We only estimate  $\alpha$  (azimuth) and  $\beta$  (elevation), the source direction.

### **TDOA Estimation Between Microphone Pairs**

• The TDOA between two microphone pairs can be estimated using cross-correlation (CC), normalized cross-correlation (NCC), generalized cross-correlation (GCC), for example:

$$Q_{ij}(t') = \mathsf{NCC}(x_i(t), x_j(t-t'))$$

 For a known sound direction (α, β), one can estimate the corresponding TDOA:

$$au_{i,j} = \operatorname*{argmax}_{t'} Q_{ij}(t')$$

### A More General Sound Propagation Model

These  $(\alpha, \beta) \leftrightarrow \tau$  input-output values do not necessarily verify the direct-path propagation model:

$$\tau_{i,j} \stackrel{?}{=} \frac{1}{\nu} \left( \| \boldsymbol{S}(\alpha,\beta) - \boldsymbol{M}_i \| - \| \boldsymbol{S}(\alpha,\beta) - \boldsymbol{M}_j \| \right)$$

More generally, we seek a function *f* that **verifies** such a relationship between known input and known output:

$$\tau = f(\alpha, \beta)$$

# **Session Summary**

- Nao has four microphones
- The microphones capture noise coming form the electronics inside the head
- The head plays the role of an acoustic filter
- The propagation model is approximated by a linear regression function

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### Learning a Regression Function

• This problem of finding *f* in

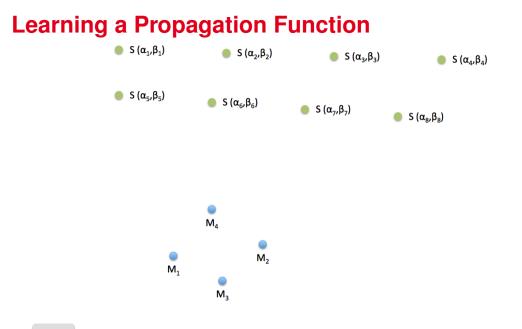
$$\tau = f(\alpha, \beta)$$

falls in the category of supervised learning.

•  $f : \mathbb{R}^2 \to \mathbb{R}$  can be **learned** by **linear regression**, for example:

$$\tau = \mathbf{a}\alpha^2 + \mathbf{b}\beta^2 + \mathbf{c}\alpha + \mathbf{d}\beta + \mathbf{e}$$

• The parameters (*a*, *b*, *c*, *d*, *e*) characterize the regression and they must be estimated.



### Learning a Propagation Function

The source is placed in K different source directions, and for each direction the TDOA is estimated.

We collect **input-output** data:  $\{\tau_k, \alpha_k, \beta_k\}_{k=1}^{k=K}$ :

$$\tau_{1} = a\alpha_{1}^{2} + b\beta_{1}^{2} + c\alpha_{1} + d\beta_{1} + e$$
  

$$\vdots$$
  

$$\tau_{k} = a\alpha_{k}^{2} + b\beta_{k}^{2} + c\alpha_{1} + d\beta_{k} + e$$
  

$$\vdots$$
  

$$\tau_{K} = a\alpha_{K}^{2} + b\beta_{K}^{2} + c\alpha_{K} + d\beta_{K} + e$$

# **Learning a Propagation Function**

Matrix notation,  $Ay = \tau$  with:

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_1 \\ \vdots \\ \tau_k \\ \vdots \\ \tau_K \end{pmatrix} \quad \boldsymbol{y} = \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \quad \boldsymbol{A} = \begin{pmatrix} \alpha_1^2 & \beta_1^2 & \alpha_1 & \beta_1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_k^2 & \beta_k^2 & \alpha_k & \beta_k & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_K^2 & \beta_K^2 & \alpha_K & \beta_K & 1 \end{pmatrix}$$

#### **Estimation of the Parameters**

This matrix-vector equation has a least-square formulation:

$$\min_{\boldsymbol{y}} \|\boldsymbol{A}\boldsymbol{y} - \boldsymbol{\tau}\|^2$$

The derivative of this error function vanishes for:

$$\mathbf{A}^{ op} \mathbf{A} \mathbf{y} = \mathbf{A}^{ op} \mathbf{\tau}$$
  
 $\mathbf{y} = \left(\mathbf{A}^{ op} \mathbf{A}\right)^{-1} \mathbf{A}^{ op} \mathbf{\tau}$ 

# **Least-square Solution**

The regression parameters are obtained with:

$$\begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} = (\mathbf{A}^{\top}\mathbf{A})^{-1}\mathbf{A}^{\top} \begin{pmatrix} \tau_{1} \\ \vdots \\ \tau_{k} \\ \vdots \\ \tau_{K} \end{pmatrix}$$

# **Session Summary**

- The regression function can be learnt from input-output data
- The source is placed in known positions (output)
- For each position a TDOA is estimated (input)
- The model parameters are estimated from these input-output pairs

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### Link Between Sound Direction and Correlation

• For each microphone pair,  $m_i$  and  $m_j$ , a propagation function is learnt:

 $\tau_{i,j} = f_{ij}(\alpha,\beta)$ 

• Combine this relationship with cross-correlation:

$$Q_{ij}(t') = \mathsf{NCC}(x_i(t), x_j(t-t'))$$

• This tells us that there is a correlation value for a sound direction:

$$Q_{ij}(\alpha,\beta) = \mathsf{NCC}(x_i(t), x_j(t - f_{i,j}(\alpha,\beta)))$$

### An Alternative to Sound-Source Direction Estimation

- **Offline learning:** Estimate the regression functions *f<sub>ij</sub>* for each microphone pair
- **Online prediction:** Estimate the direction of the sound by combining the cross-correlation functions between all microphone pairs and for all possible directions.

### Learning a Sound Propagation Model

For each microphone pair *i* and *j*:

- 1. Place a sound source in known directions  $(\alpha_1, \beta_1), \ldots, (\alpha_k, \beta_k), \ldots, (\alpha_K, \beta_K)$ .
- **2**. For each sound source in direction  $(\alpha_k, \beta_k)$  estimate the TDOA with:

$$au_k = \operatorname*{argmax}_{t'} \operatorname{NCC}(x_i(t), x_j(t-t'))$$

- **3**. Collect a set of *K* input-output pairs  $\tau_k \leftrightarrow (\alpha_k, \beta_k)$ .
- Estimate the parameters of the linear regression f<sub>ij</sub>, namely a<sub>ij</sub>, b<sub>ij</sub>, c<sub>ij</sub>, d<sub>ij</sub>, e<sub>ij</sub>.

# **Predicting a Sound Direction**

A method to find an unknown direction (azimuth  $\alpha$  and elevation  $\beta$ ) of a sound source:

- 1. Discretize the space of possible directions.
- **2**. For each direction  $(\alpha, \beta)$  and for each microphone pair  $(m_i, m_j)$ , compute:

$$Q_{ij}(\alpha,\beta) = \mathsf{NCC}(x_i(t), x_j(t - f_{i,j}(\alpha,\beta)))$$

**3**. The  $(\alpha, \beta)$  values that optimize the correlation over all the microphone pairs is the estimated sound direction:

$$\hat{\alpha}, \hat{\beta} = \operatorname*{argmax}_{\alpha, \beta} \sum_{i \neq j} Q_{ij}(\alpha, \beta)$$

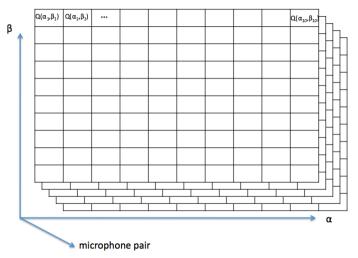
# **Correlation Map for One Microphone Pair**

Q(α <sub>1</sub> ,β <sub>1</sub> )	Q(α <sub>2</sub> ,β <sub>2</sub> )				Q(α <sub>10</sub> ,β <sub>1</sub>

α

β

# **Correlation Map for Many Microphone Pairs**



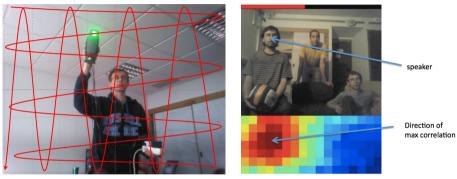
# **Session Summary**

- Practical method for estimating the TDOA
- The linear regression parameters are estimated for each microphone pair
- The cross-correlation is estimated for each direction and for each pair
- The cross-correlations from all the pairs are summed up
- The sound-direction corresponds to the best cross-correlation

# 3. Sound-Source Localization

- 1. Time difference of arrival (TDOA)
- 2. Estimation of TDOA by cross-correlation
- 3. Estimation of TDOA in the spectral domain
- 4. The geometry of two microphones
- 5. Direction of arrival
- 6. Using more than two microphones
- 7. Embedding the microphones in a robot head
- 8. Learning a sound propagation model
- 9. Predicting direction of a sound with a robot head
- **10. Example of sound direction estimation**





Learning

Prediction

### **Nao Localizes Speech**

INSERT sound-direction-correlation.avi

# **Session Summary**

- The method was used to estimate a propagation model for NAO
- This model can then be used to estimate the direction of a speaking person
- The estimated direction corresponds to the maximum of the cross-correlation across all the microphone pairs

# **Week Summary**

- The time difference of arrival between a microphone pair can be estimated in the temporal domain or in the frequency domain.
- General cross-correlation is a flexible tool to estimate the time difference of arrival.
- With two or more microphones it is possible to find the sound-source position in 3D.

# Week Summary (Continued)

- The direct propagation equations lead to solving a non-linear optimization problem.
- It is possible to adopt a supervised learning approach that replaces the propagation model with a linear regression function.
- This yields a methodology for sound-direction estimation with a microphone array embedded in a robot head.